MATHEMATICAL WORK OF A FUTURE TEACHER IN TEACHING THE BOX PLOT DIAGRAM

Paula Verdugo-Hernández¹
Gonzalo Espinoza-Vásquez²
Patricio Cumsille³

ABSTRACT

Objective: The study aims to characterize the teaching proposals of the future Mathematics teacher in the context of their practical training in the final year of their university studies.

Theoretical Framework: To achieve this, we consider the theory of Mathematical Workspaces, which allows for the analysis of both the mathematical activity that an individual engages in while solving a mathematical task, and the activity that is promoted during teaching.

Method: A qualitative methodology is adopted through the design of an instrumental case study. The case pertains to a future teacher conducting a class on constructing a box plot. This class was observed and transcribed for analysis in light of the proposed mathematical work.

Results and Discussion: The mathematical work exhibited by the future teacher includes a strong semiotic component and the use of non-material artifacts for quartile calculations. Students' prior knowledge is utilized in this context, with procedural aspects taking precedence over statistical thinking.

Implications of the Research: The study raises concerns about the statistical education of mathematics teachers and its impact on future teaching proposals.

Originality/Value: This research contributes to the study of statistics and initial teacher training in their influence on the future practices of Mathematics teachers. It provides a characterization of the mathematical work promoted by a future teacher and offers insights into concerns regarding the development of statistical thinking.

Keywords: Secondary Education, Pedagogical Practice, Mathematical Work, Statistical Graphs, Box Plot Diagram.

TRABALHO MATEMÁTICO DE UM FUTURO PROFESSOR NO ENSINO DA DIAGRAMA DO CAIXA

RESUMO

Objetivo: O estudo procura caracterizar as propostas de ensino do futuro professor de Matemática no contexto da sua formação prática no último ano de estudos universitários.

¹ Campus Linares, Escuela de Pedagogía en Ciencias Naturales y Exactas, Facultad de Ciencias de la Educación, Universidad de Talca, Linares, Chile. E-mail: paulasinttia@gmail.com
Orcid: https://orcid.org/0000-0001-6162-654X

² Universidad Alberto Hurtado, Facultad de Educación, Departamento de Pedagogías Medias y Didácticas Específicas, Santiago, Chile. E-mail: gespinosa@uahurtado.cl
Orcid: https://orcid.org/0000-0003-4500-4542

³ Departamento de Ciencias Básicas, Facultad de Ciencias, Universidad del Bío Bío, Campus Fernando May, Avenida Andrés Bello 720, Casilla 447, Chillán, Chile. E-mail: pcumsille@ubiobio.cl
Orcid: https://orcid.org/0000-0003-1067-129X

Referencial Teórico: Para isso consideramos a teoria dos Espaços Matemáticos de Trabalho, que permite a análise tanto da atividade matemática que um indivíduo desenvolve ao resolver uma tarefa matemática quanto daquela atividade que é promovida para/durante o ensino.

Método: Adota-se uma metodologia qualitativa, através do desenho de um estudo de caso instrumental. O caso corresponde a uma futura professora que ministra uma aula sobre a construção do diagrama de caixa e bigode. Essa aula foi observada e transcrita para análise à luz do trabalho matemático proposto.

Resultados e Discussão: O trabalho matemático realizado pelo futuro professor inclui uma forte componente semiótica e a utilização de artefactos não materiais para o cálculo de quartis. Nessas construções são utilizados os conhecimentos prévios dos alunos, predominando os aspectos processuais sobre o pensamento estatístico.

Implicações da pesquisa: O estudo levanta alertas sobre a formação estatística de professores de matemática e seu impacto nas futuras propostas de ensino.

Originalidade/Valor: Esta investigação contribui para o estudo da estatística e da formação inicial de professores no seu impacto nas práticas futuras dos professores de Matemática. A pesquisa oferece uma caracterização do trabalho matemático que promove um futuro professor e fornece elementos para reflexão sobre preocupações relativas ao desenvolvimento do pensamento estatístico.


TRABAJO MATEMÁTICO DE UN FUTURO PROFESOR EN LA ENSEÑANZA DEL DIAGRAMA DE CAJA Y BIGOTES

RESUMEN

Objetivo: El estudio busca caracterizar las propuestas de enseñanza del futuro profesor de Matemática en el contexto de su formación práctica de su último año de carrera universitaria.

Marco Teórico: Para ello, consideramos la teoría de los Espacios de Trabajo Matemático, que permite el análisis tanto de la actividad matemática que desarrolla un individuo al resolver una tarea matemática como aquella actividad que se promueve para/durante la enseñanza.

Método: Se adopta una metodología de tipo cualitativo, a través del diseño de un estudio de caso instrumental. El caso corresponde a un futuro profesor que imparte una clase sobre la construcción del diagrama de caja y bigotes. Esta clase se observó y transcribió para su análisis a la luz del trabajo matemático propuesto.

Resultados y Discusión: El trabajo matemático desplegado por el futuro profesor contempla un fuerte componente semiótico y el uso de artefactos no materiales para el cálculo de cuartiles. Se utilizan los conocimientos previos de los estudiantes en dicha construcción, predominando aspectos procedimentales sobre el pensamiento estadístico.

Implicaciones de la investigación: El estudio levanta advertencias sobre la formación estadística de los profesores de matemática y su incidencia en las futuras propuestas de enseñanza.

Originalidad/Valor: Esta investigación aporta al estudio de la estadística y a la formación inicial docente en su impacto sobre las futuras prácticas de los profesores de Matemática. La investigación ofrece una caracterización del trabajo matemático que promueve un futuro profesor y brinda elementos de reflexión sobre las preocupaciones respecto del desarrollo del pensamiento estadístico.

Palabras clave: Enseñanza Secundaria, Práctica Pedagógica, Trabajo Matemático, Gráficos Estadísticos, Diagrama De Caja Y Bigote.

RGSA adota a Licença de Atribuição CC BY do Creative Commons (https://creativecommons.org/licenses/by/4.0/).
1 INTRODUCTION

Statistics is fundamental in all fields of study and for society in general (e.g. Salcedo and Díaz-Levicoy, 2022). We agree with Estrella (2017), who declares Statistics as one of the most important civil rights today, since they contribute to quality education and equity. Thus, statistical literacy is key for every individual to acquire at school the ability to understand and use this discipline in their daily life, for example, in making informed decisions.

Research such as that of Batanero and Borovcnik (2011) shows multiple difficulties that arise in Statistics and Probabilities (e.g. Rodríguez-Alveal and Díaz-Levicoy, 2021; Machuca and Montoya-Delgadillo, 2022) or in the interpretations of statistical graphs (Díaz-Levicoy et al., 2021, Souza and Monteiro, 2020), to mention a few.

One of the specific difficulties arises when constructing and interpreting the box and whisker diagram. For example, the use of the median, since it is not perceived as a measure of central tendency by students. Furthermore, the width of the box is confused with the dispersion of the data (Bakker et al., 2004) and the areas of the boxes with the amount of data (Lem et al., 2012; 2013), that is, while The larger one of them is, the greater the amount of data it has.

Studies such as those by Ben-Zvi and Makar (2016) and Crites and Laurent (2015) suggest that teaching in this field begins from the initial courses, in order to form statistically literate individuals. In this sense, it is necessary to pay attention to those who teach Statistics and their training. This line has promoted various research that shows the importance of focusing on teacher training (e.g. Bruns and Luque, 2014; Jiménez et al., 2014; Pino-Fan et al., 2022). In fact, Bruns and Luque (2014) point out that one of the most relevant factors for the training of students is the quality of the teachers. Indeed, government demands on universities, the results in different tests and the teaching itself require constant attention to teacher training, above all, attention to what really happens in the classroom.

There is little research that addresses the understanding of the box and whisker graph in teachers in training or in-service teachers (Gutiérrez-Martínez et al., 2024). Among them, the research by Verdugo-Hernández and Espinoza-Vásquez (2023) characterizes the mathematical work proposed by a future teacher (FP) in their last year of training, in the context of a pandemic. In this study, it shows how the FP seeks to develop the mathematical skills contemplated in the Chilean curriculum (Ministry of Education [MINEDUC], 2016) during a class on position measurements (quartiles). There it is observed that such work contemplated different skills than those planned by the FP.

This report addresses the line of practical training (or practicum) in initial teacher
training (FID) as a context for future teachers' first approaches to teaching. The objective is to characterize the teaching proposals of mathematics FP in the context of its practical training. Specifically, we seek to contribute to the field of initial training of mathematics teachers, from the perspective of the role that the FP will play, asking ourselves: What is the mathematical work that the future teacher proposes when teaching the box and whisker diagram?

2 THEORETICAL FRAMEWORK

To address this question, we consider the model of Mathematical Work Spaces (ETM, Kuzniak et al., 2022), which aims to study mathematical work in a specific educational context, in order to promote the teaching and learning of mathematics through the approach and resolution of mathematical tasks, which allows observing certain aspects of the future teacher's teaching proposals.

The ETM model (figure 1) is organized by two main planes, which in turn have three components each. The epistemological plane, with the referential components, representations and artifacts, and the cognitive plane, with visualization, construction and testing. The components are articulated with each other through three genesis: semiotic (based on the registers of semiotic representation that ensures the tangible objects of the ETM their status as operational mathematical objects), instrumental (which allows the artifacts to be made operational in the process). constructive) and discursive (which gives meaning to the properties to put them at the service of mathematical reasoning).

Figure 1
Outline of the Mathematical Workspace

Source: Kuzniak et al. (2022)

The ETM considers three types of Workspaces: reference, ideal and personal. In this
study we focus on the ideal ETM, which is directly related to that space that allows work in a school institution with a specific educational purpose. In the ideal ETM, a distinction is made between potential and current (or implemented) ETM (Henríquez-Rivas et al., 2022). The distinction is made between what is proposed versus what actually happens during teaching, respectively. For its part, personal ETM can be from both the teacher and the student and is related to the way in which they approach and solve certain mathematical tasks. This ETM is influenced by the individual experiences, knowledge and strategies of the subject. Here the importance of the uniqueness of each individual in the educational process is recognized along with their own understanding of the mathematical notions involved.

In terms of this theoretical framework, the objective translates into characterizing the ideal mathematical work implemented by a teacher in training in his professional practice for teaching the box and mustache diagram.

3 METHODOLOGY

To address the objective and question, we adopted an interpretive paradigm with a qualitative methodology (Denzin and Lincoln, 2000). The research is designed as an instrumental case study (Stake, 2007) in which it studies the teaching proposal of a future mathematics teacher (FP), who we will call Martina. She is in her last semester of the Pedagogy degree in Secondary Education in Mathematics, at a Chilean university. Martina takes the subject of professional practice, which requires her to complete 30 hours a week in an educational establishment, for approximately one semester.

The practice center assigned to Martina corresponds to a municipal (public) high school in the State of Chile. The center has between 40 to 45 students per course. Martina does part of her practice hours in a first year course (14 to 15 years old), where she is accompanied and supervised by a teacher (guide) from the high school and a tutor from the university where she is studying.

Martina practices it in person at the high school, having to complete an initial period of adaptation and observation; plan the classes to be taught; implement planned classes; participate in meetings with a guide teacher and tutor and prepare a written report with the activities developed during her professional practice.
3.1 SELECCIÓN DEL CASO

Martina has been selected because she is taking the professional practice subject and planning a session on Statistics topics. This obeys the criterion of convenience (Creswel, 2014) and accessibility to the case (Loughran et al., 2008). On this occasion we will present the analysis of the first class of the unit, since it deals with the notions that are the subject of our interest and presents common characteristics in relation to the structure of the other classes (Coyne, 1997).

3.2 DATA COLLECTION AND ANALYSIS

The data comes from the video recordings of the classes and the information requested from Martina regarding teaching planning or pedagogical resources that she had available. This report considers the (non-participant) observation of the class session implemented by Martina, where she addressed the construction of box and whisker plots.

The analyzes use a methodology based on Kuzniak and Nechache, (2021) and Henríquez-Rivas and Verdugo-Hernández, (2023) to describe the mathematical work of the class. The work moments of the FP are identified, and then interpreted in terms of the ETM. Table 1 shows the analysis protocol used.

Table 1
Protocol for the analysis related to the ETM associated with the box plot.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Components</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semiotic Genesis (GS)</td>
<td>Representative</td>
<td>Relate objects from the box plot and their significant elements.</td>
</tr>
<tr>
<td>Display</td>
<td>Display</td>
<td>Interprets and relates the objects of the box and mustache diagram according to cognitive activities related to the records of semiotic representations (identification, treatments, conversions).</td>
</tr>
<tr>
<td>Instrumental Genesis (GI)</td>
<td>Artifact</td>
<td>Use material type artifacts or a symbolic system.</td>
</tr>
<tr>
<td>Construction</td>
<td>Construction</td>
<td>It is based on the processes given by the actions triggered by the artifacts used and the use techniques associated with the box diagram.</td>
</tr>
<tr>
<td>Discursive Genesis (GD)</td>
<td>Referential</td>
<td>Use definitions, properties or theorems related to the box plot.</td>
</tr>
<tr>
<td>Proof</td>
<td>Proof</td>
<td>Discursive reasoning is based on different forms of justification, argumentation or boxplot demonstration.</td>
</tr>
</tbody>
</table>

Source: Adapted from Henríquez-Rivas and Verdugo-Hernández, (2023).

3.3 CLASS DESCRIPTION: MOMENTS AND EPISODES

To organize the data and structure the analysis, the three moments of the class were considered: beginning, development and closing, for which it was possible to identify as the
objective “Understand the construction process of the box and whisker diagram”, as seen in Figure 2. Each moment contemplates one or two episodes. In the Beginning, the objective is presented and some knowledge is evoked that will help construct the box and whisker plot (episode 1). In this reminder, Martina includes new concepts, such as extreme values or outliers. In Development, the calculation of quartiles for a data set and the construction of a box and whisker plot are identified (episode 2). Furthermore, Martina proposes to analyze the distribution of the data through the diagram (episode 3). For Closing, a task is proposed on the construction of the box and whisker plot (episode 4), called the exit ticket.

Figure 2
Session planning

<table>
<thead>
<tr>
<th>Momentos de la clase</th>
<th>Desarrollo (40 min)</th>
<th>Cierre (10 min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inicio (10 min)</td>
<td>La profesora socializa con los estudiantes respecto a uno de los temas contenidos a aprender, preguntando si han escuchado hablar del diagrama de caja y bigotes. Luego, explica algunos de los conceptos más importantes para poder entender el diagrama, siete algunos ya conocidos de estos ya aprendidos en la unidad anterior. Los conceptos son algunos como cuartiles, rango intercuartílico, mediana, límite superior e inferior y valores extremos.</td>
<td>Con la finalidad de cerrar la clase verificando si el contenido fue aprendido por los estudiantes, se les entrega un ticket de salida que presenta conjuntos de datos, respecto a los cuales los estudiantes deben construir el diagrama de caja y bigotes correspondientes. Cabe destacar que los tickets de salidas en 4 formas distintas, es decir, hay variedad de conjuntos presentados para que  cada estudiante y sus compañeros ubicados cerca tengan un ejercicio diferente. Finalmente, la profesora recoge los tickets de salida para su posterior revisión y retroalimentación.</td>
</tr>
<tr>
<td></td>
<td>Se presenta un ppt que muestra la definición de cada uno de los conceptos, incluyendo la explicación de cuál es la finalidad del diagrama de caja y bigotes, para luego dar paso a la explicación de cómo construir este diagrama paso a paso a través de un ejemplo en la pizarra.</td>
<td></td>
</tr>
</tbody>
</table>

4 RESULTS AND DISCUSSIONS

Next, we present the results of the session analysis, organized by episodes.

4.1 EPISODE 1: PRESENTATION OF THE OBJECTIVE

Martina points out that the class will be about the box and whisker plot. She asks if “anyone has heard of it,” to which the students respond “no.” Martina states the objective of the class as “understanding the process of constructing the box and mustache diagram.” She thereby sheds light on the work she seeks with her students: constructing such diagrams. The objective proposes a procedural work, using prior knowledge of statistics and instruments for this construction.

The FP highlights some important concepts, such as quartiles and median, whose definitions she completes on the board, as expected in the planning (figure 2), in response to the students. Martina points out that she will introduce new concepts such as outliers or extreme values, providing her definition as seen in figure 3 and in the following extract:
M: For example, if I have several pieces of data that are in a row, two, three, four, and I have a value that is 15, for example, that value is going to be an extreme value. And we are also going to know the formulas that allow us to identify which are the extreme values and which are not.

This battery of concepts seeks to expand the referential component of the ETMi and support the construction of the diagram. Martina defines the upper and lower limits for the construction of the graph as “values that will mark the outliers of a data distribution.” She thereby establishes the procedure for determining such values as an artifact in the construction of the diagram. She then allows time for students to record these concepts in their notebooks before moving on to the box-and-whisker plot.

The diagram (Figure 3) includes the representation of quartiles, median, limits and extreme values. Here the semiotic genesis is activated, from the referential, through the location in the diagram of the newly defined or remembered concepts. Martina proposes a definition of the diagram (to establish it as part of the students' reference), without mentioning the reference axes, the calculation of the minimum and maximum or the extreme values. With this, the semiotic work points to the iconic, except for the clarification on the type of variables or statistical data that can be represented in this type of graph.

Figure 3
Box and whiskers representation

Martina activates students' prior knowledge, necessary for the study of this graph, mobilizing the students' personal ETM, as seen below:

M: It is a graph for quantitative variables. Remember the quantitative variables, what was that?
E: The ones that were counting, right?
M: Yes. The counting variables are numerical variables, right? So, this graph will allow us to see what the distribution of the data is like, why? It indicates here the first quartile,
the second quartile which corresponds to the median and the third quartile, which also indicates the extreme values, which are the ones that I told you are furthest from the general frequencies of the data and tells us also a minimum value and a maximum value.

From the perspective of the ETM, the FP evokes elements of the reference of variables and statistical data, in addition to the relationship with position measures. It aims to support the construction of the diagram based on the properties of the objects involved, structuring the ideal ETM. Martina highlights the elements of the diagram and asks students to take note of them. Furthermore, she anticipates that this same diagram will be studied with a number line to indicate the quartiles and the maximum and minimum values, not yet defined.

4.2 EPISODE 2: BOX AND WHISTAKE PLOT CONSTRUCTION

In this episode, FP proposes a data set (Figure 4) to calculate quartiles. The data are fictitious ages of a group of people. Martina emphasizes that the objective is to build the diagram for this contextualized data. The proposed work starts from the numerical record and makes use of the formula for calculating quartiles.

Figure 4
Age data of university students

The data are presented in order to determine the position of the quartiles. Here an interaction is observed between the semiotic genesis (GS), through the quartiles as separators of the sample in blocks, and the discursive genesis (GD), since its calculation allows these blocks to be determined. This calculation occurs using a formula, as seen below:

M: We have ten pieces of information, right? That means that n is 10. To construct the box, step 1 is to order the data from smallest to largest, just as we have been doing during the semester, in all the measurements that we have calculated. Are they ordered?
Is if
M: The second step is to calculate the 3 quartiles. We know how to do that, right? what is the formula?
E2: n times K divided by 100
M: where n is the amount of data and k is?
It is: the percentile
M: [...]So, quartile 1 corresponds to 19, quartile 2 to 20 because the average between 20 and 20 is 20 and quartile 3 to 23, what are we going to do to start building this diagram?
The formula constitutes a symbolic artifact for the construction of the diagram, showing an attempt to activate the instrumental genesis (IG) from the referential component. The excerpt shows how students follow the class and know the formula, applying it in each quartile. The proposed circulation in the ETMi is: GS → referential → GI

Martina continues with the calculation of the C1, C2 and C3 quartiles. The students correctly carry out the calculations orally and are able to locate the corresponding data (part a in figure 5). The semiotic work is articulated with the properties of the quartiles and is reinforced by including the number line to locate them, referring to the complete set of data. Figure 5 shows how this part of the work is organized. In b) the use of the formula and the adaptation of the result obtained (approximation of 2.5 to 3) is observed to determine the position of the C1 quartile together with determining its value by counting the data provided in a).

Martina uses the students’ knowledge to determine the quartiles and to structure her ETMi. However, there is no evidence in Martina’s reports that she has tried said formula. At the same time, neither Martina nor the students question the indicated formulas, they only apply them to solve the task. From these calculations, Martina points out that she will draw the box of the diagram. She positions the 1st and 3rd quartiles on the number line and asks her students about the value that goes in between them, to which the students respond “the median.” This reflects other prior knowledge that underpins the ETMi. Martina specifies that she does not always go right in the middle, without further arguments and evoking properties of the median. Once again, the relationship between the semiotic genesis (“goes in the middle”) and the referential component (properties of the median) is observed.

Subsequently, Martina writes how the calculation of the lower and upper limit should be carried out (figure 5; c and d), without arguing about the formula. Although the activation of the referential is evident, there is no presence of arguments in her speech, which prevents observing the component of the evidence or the discursive genesis.
The formulas to calculate the lower and upper limits (figure 5; c and d) are related to the normal distribution, which is very important in Statistics. Martina uses the quartile values to determine both limits.

M: Let's calculate the admissible limits. That is, a lower limit and an upper limit. What this limit marks for us is where the mustache can be. How far can it be? [...] It's not that it goes in the mustache. The mustache is not always going to end at the limit, it can end sooner, right? So the inner limit formula would be 1.5, sorry. Quartile 1 minus 1.5 times the interquartile range, how much would that leave us? Calculate it in your notebook.

The final plot (Figure 5, e and f) includes the quartiles, lower and upper limits, and interquartile range. Martina points out that, based on the interquartile range, she will be able to calculate both limits. In addition, she explains that they mark the length of the mustache. As in the calculation of quartiles, the formula to determine the interquartile range takes on the role of artifact in the construction of the diagram.

The articulation of all these elements shows the circulation in the ETMi: referential → GI → GS → referential. The FP seeks, from the reference, to build the diagram by activating the instrumental genesis; The diagram seeks to establish a new representation of the data and its distribution, supported by the determination/location of the quartiles, activating the GS. Martina positions the diagram and its properties as a new object in the reference as a return to the reference in the EMTi.
Regarding semiotic work, Martina performs a conversion, in Duval's sense, of the data to bring it to the diagram. Here a numerical treatment is observed (figure 5 a), b), c) and d)) through the calculation of quartiles and the interquartile range. The new graphic representation (in (e) and f)) materializes the objective of the class and allows the distribution of the data to be visualized, specifying the ETMi. Martina calculates both limits and explains their meanings.

M: What does this tell us? [lower limit] It will tell us that the lower limit, that is, all the data that is smaller than this limit, which is 13. We are going to place 13 very far away [ Indicating with the left hand the position on the number line], whichever is smaller, that is, all those that are towards there, will be extreme values. In this case, are there extreme values for that side? Untrue? Because number 18 is the smallest. At the upper limit it is 29, that is, here is the upper number. Do we have extreme values? Is if.
M: Which one? We have here the 32, right? Because 29 reaches right at the upper limit. 32 is an extreme value. What are we going to do with that? Let's build the mustaches.

Martina points out that the lower and upper mustache are determined by the respective limits. These guidelines for students meet the objective of the session and expand the reference intended in the ETMi by defining and applying the new concepts. The choice of data set appears to be intentional to highlight outliers and boundaries in relation to the quartiles and plot. Martina recognizes data 32 as an extreme value (part a of figure 5). The FP points out that values such as 32 allow for an interpretation of the upper limit and the concept of atypical value, as part of the referential component. This episode ends with an explanation of each step carried out in the construction of the diagram (figure 6), focusing its explanation on extreme cases.

**Figure 6**
*Steps to carry out to construct the box and mustache diagram.*
4.3 EPISODE 3: DATA DISTRIBUTION ANALYSIS IN THE DIAGRAM

In this episode, the FP presents types of distribution based on the shapes of the diagrams (figure 7), where it includes a characteristic of each case. From these examples, Martina establishes an interpretation of the data and a way of reading the diagram, which corresponds to a work from the GS to the reference, to build the concept of distribution.

M: Let's see the types of symmetrical distribution that, as the name suggests, both, let's say with mini boxes, are going to be the same size, they are going to be symmetrical, which happens when the median is right in the middle. The second mini box is going to be bigger. And what does that tell us? That the values are more separated, that is, they are more dispersed. The values from the median onwards. For example, this diagram, what type of symmetry or asymmetry would it have? Negative asymmetry that when it is repeated before the median, the one before it is larger, that is, when the smaller values are more dispersed.

Martina seeks to strengthen the epistemological plane of reference through this interpretation. With this, the proposed work shows a change in the directionality of activation of the ETMi, from the GS to the reference, unlike the work in the previous episode.

4.4 EPISODE 4: EXIT TICKET

To finish the class, the FP gives the students different sets of data and asks them to build the corresponding diagram. Students must replicate the procedures shown during the session (Figure 6). Martina points out that the activity seeks to demonstrate what the students have achieved. In this way, the task proposes the same mathematical work and circulation of episode 2, based on the properties of the quartiles (referential), the instrumental genesis to construct the
Mathematical Work of a Future Teacher in Teaching the Box Plot Diagram

diagram and the corresponding semiotic work. Since Martina does not request the interpretation of the statisticians or the construction process, there is an absence of the GD in this task.

5 DISCUSSION AND CONCLUSIONS

Martina begins by remembering definitions and explains the new new concepts (interquartile range, the lower and upper limits) to arrive at the construction of the box and whisker plot. This limits student participation in the learning process. Despite Martina's conventionality in her class, it is important to recognize that the interaction with the students was able to counteract possible limiting effects of her beginning. Martina encourages interaction with his students, contrary to the case studied by Verdugo-Hernández and Espinoza-Vásquez, (2023). This can be explained given that the class studied was held via Zoom, during the pandemic. However, it would be interesting to study what factors can contribute to developing a more participatory class than another.

In the work that Martina proposes, we observe a connection between the calculations he performs and the construction of the diagram, evidencing the presence of semiotic genesis, related to signs and their meaning. In this case, the symbols and statistical calculations are related to their graphic representation, emphasizing the importance of the reference in its teaching, which is distinctive of its ETMi.

On the other hand, by not including arguments or evidence around the definitions, Martina limits students' ability to fully understand the box concepts involved. The absence of the GD in the session prevents students from being able to contextualize the box plot, without the opportunity to question, justify or validate the definitions and concepts presented.

This coincides with the results of Verdugo-Hernández and Espinoza-Vásquez (2023), where the formulas to determine the quartiles are not questioned. This is observed when calculating quartile 1 and approximating 2.5 to 3. In this case it could be calculated as the average between data #2 and #3 of the ordered data. In that sense, we consider it convenient to avoid ambiguities in the use of the formulas, providing an algorithm or method to determine the quartiles in a unique way.

Furthermore, by not encouraging discussion around the definitions, for example, of symmetric and asymmetric distribution and their applicability to compare different distributions, an opportunity to promote statistical thinking is lost (e.g. Copur-Gencturk, 2015). There is also little interpretation of the diagram regarding its usefulness in understanding the distribution of the data. This moves away from the development of statistical thinking or the
sense of data beyond the numerical. We notice an environment that lacks statistical learning in its broad spectrum (Garfield and Ben-Zvi, 2008). In this line, the discursive genesis, involved in the construction of arguments, is fundamental for the development of conceptual understanding and problem solving in Statistics. Their absence can also negatively impact your understanding and mastery of statistical concepts. It has also been reported in practicing teachers (Salcedo and Díaz-Levicoy, 2022).

It is observed that Martina's class tends to follow a traditional approach, the same phenomenon found in Verdugo-Hernández and Espinoza-Vásquez (2023), where the emphasis falls on the transmission of information by the FP, biased towards a statistical-mathematical (Burrill, 2006). Martina's practice reflects a way of working with Statistics in secondary education and in mathematics class. As Stohl (2005) points out, it is important to review teacher training, not only in their programs, but in what they actually do in the classroom. For example, in the possibilities of including technology as a resource for teaching (Batanero and Borovcnik, 2016), fundamental in today's society and that both the study by Verdugo-Hernández and Espinoza-Vásquez (2023) and the present work indicate that Future teachers do not integrate educational software in their statistics classes, which suggests the need for greater attention to this aspect in teacher training.

Finally, this study is an advance in the characterization of the teaching proposed by future teachers, however, we consider that the review of such teaching practices remains an open topic, which requires research efforts to support the training of Mathematics teachers who teach statistics.

ACKNOWLEDGMENT


REFERENCES


Profesorado para Enseñar Estadística: Retos y Oportunidades (pp. 3-20). Centro de Investigación en Educación Matemática y Estadística. Universidad Católica del Maule.


