USING ANALYSIS OF VARIANCE AND TUKEY'S TEST TO COMPARE FOUR BRANDS OF INDUSTRIAL PAINT

Daniel Marques de Deus
Cleginaldo Pereira de Carvalho
Felipe Cury Mazza
Sílvio Sérgio Silveira de Siqueira
João Ercio Miranda Junior
Nilo Antonio de Souza Sampaio

ABSTRACT

Objective: The aim of this paper is to show the application of Analysis of Variance and Tukey's Test to compare four brands of paint used in a company's production.

Theoretical framework: Analysis of Variance, or ANOVA, is a statistical method used to determine whether there are significant differences between the means of three or more independent groups. This technique was developed by the British statistician and geneticist Ronald Fisher at the beginning of the 20th century (Bosco & Bispo, 2010).

Method: Data was taken from a company in the South of Rio de Janeiro and a case study was carried out using Analysis of Variance and Tukey's test.

Final Considerations: This result indicates that the hardness of the paint mixtures differs significantly. Some of the group averages are known to be different. Tukey's comparison results were used to formally test whether the difference between a pair of groups is statistically significant. It was found that Paint 4 has a higher hardness than the others.

Implications of the research: The use cases of Analysis of Variance and Tukey's Test are multiplying in the scientific literature and are proving to be highly effective for processing data in a wide variety of areas.

Originality/value: Despite being well-known statistical tools, Analysis of Variance and Tukey's Tests are only valid in cases where there are two types of samples in which the statistical assumptions have been confirmed.

Keywords: Analysis of Variance, Tukey Test, Anderson-Darling Test, Normalized Data.
RESUMO

Objetivo: O objetivo deste trabalho é mostrar a aplicação da Análise de Variância e do Teste de Tukey para comparação de quatro marcas de tinta para ser utilizada na produção de uma empresa.

Referencial teórico: A Análise de Variância, ou ANOVA, é um método estatístico utilizado para determinar se há diferenças significativas entre as médias de três ou mais grupos independentes. Essa técnica foi desenvolvida pelo estatístico e geneticista britânico Ronald Fisher no início do século XX (Bosco v & Bispo, 2010).

Método: Foram retirados dados de uma empresa da Região Sul Fluminense e foi realizado um estudo de caso usando Análise de Variância e Teste de Tukey.

Considerações Finais: Este resultado indica que a dureza das misturas de tinta difere significativamente. Sabe-se que algumas das médias do grupo são diferentes. Foram utilizados os resultados da comparação de Tukey para testar formalmente se a diferença entre um par de grupos é estatisticamente significativa. Verificou-se que a Tinta 4 possui dureza maior que as outras.

Implicações da pesquisa: Os casos de utilização de Análise de Variância e Teste de Tukey se multiplicam pela literatura científica e se mostram altamente eficazes para tratar dados nas mais diversas áreas.

Originalidade/valor: Apesar de serem Ferramentas Estatísticas bastante conhecidas a Análise de Variância e os Testes de Tukey só são válidos para casos em que existem mais de dois tipos de amostras em que os pressupostos estatísticos foram confirmados.

Palavras-chave: Análise de Variância, Teste de Tukey, Teste de Anderson-Darling, Dados Normalizados.

USO DEL ANÁLISIS DE VARIANZA Y LA PRUEBA DE TUKEY PARA COMPARAR CUATRO MARCAS DE PINTURA INDUSTRIAL

RESUMEN

Objetivo: El objetivo de este trabajo es mostrar la aplicación del análisis de varianza y la prueba de Tukey para comparar cuatro marcas de pintura utilizadas en la producción de una empresa.

Fundamento teórico: El análisis de la varianza, o ANOVA, es un método estadístico utilizado para determinar si existen diferencias significativas entre las medias de tres o más grupos independientes. Esta técnica fue desarrollada por el estadístico y genetista británico Ronald Fisher a principios del siglo XX (Bosco v & Bispo, 2010).

Método: Los datos se tomaron de una empresa del sur de Río de Janeiro y se realizó un estudio de caso mediante análisis de varianza y la prueba de Tukey.

Consideraciones finales: Este resultado indica que la dureza de las mezclas de pintura difiere significativamente. Se sabe que algunas de las medias de los grupos son diferentes. Se utilizaron los resultados de la comparación de Tukey para comprobar formalmente si la diferencia entre un par de grupos es estadísticamente significativa. Se comprobó que la pintura 4 tenía una dureza mayor que las demás.

Repercusiones para la investigación: Los casos de uso del análisis de la varianza y la prueba de Tukey se multiplican en la literatura científica y están demostrando ser muy eficaces para procesar datos en una gran variedad de ámbitos.

Originalidad/valor: A pesar de ser herramientas estadísticas muy conocidas, el análisis de la varianza y las pruebas de Tukey sólo son válidas en los casos en los que hay más de dos tipos de muestras en las que se han confirmado los supuestos estadísticos.

Palabras clave: Análisis de la Varianza, Prueba de Tukey, Prueba de Anderson-Darling, Datos Normalizados.
1 INTRODUCTION

During the last century, statistics revolutionized science by presenting useful models that modernized the research process in the direction of better research parameters, making it possible to guide decision making in a wide variety of areas. Statistical methods were developed as a mixture of science and logic for the solution and investigation of problems in various areas of human knowledge (Abraão et al., 2024; da Motta Reis et al., 2023; Sampaio, Mazza, Sérgio, et al., 2024; Sampaio, Mazza, Siqueira, et al., 2024; A. C. P. da Silva et al., 2023).

Launching a new product and/or process usually involves working with a large number of variables. Proper planning of the experiments that must be used to manipulate these variables and arrive at the desired answers is indispensable for obtaining reliable results and carrying out consistent statistical analyses. In this context, it is no longer possible to develop products and processes empirically, as was done in the past. Fierce competition, the spread of technological processes and the responsibility of the scientific community now make such procedures impossible. Optimizing processes requires more than ever a robust and efficient statistical study (de Souza Sampaio et al., 2022; Espuny et al., 2023; Mazza et al., 2022, 2024; Saboia et al., 2024; Sales et al., 2021; Siqueira et al., 2024).

Mathematical modeling plays a crucial role in the field of process optimization. In fact, the effectiveness of process optimization is intrinsically linked to the quality of the models obtained. In other words, a process of optimization can only achieve satisfactory results if the mathematical model representing it is of high quality. Mathematical modeling finds utility in a wide spectrum, encompassing biology, medicine, chemistry, and the social sciences. The significance of mathematical models becomes evident in diverse applications, such as water resource management (Gomes et al., 2023).

The aim of this article is to present the results obtained with Analysis of Variance and the Tukey test for a case study in a company in the southern region of Rio de Janeiro state, Brazil.
2 THEORETICAL BACKGROUND

Conventional statistical tests are usually called parametric tests. Parametric tests are used more frequently than non-parametric tests in many medical articles, because most of the medical researchers are familiar with and the statistical software packages strongly support parametric tests. Parametric tests require important assumption; assumption of normality which means that distribution of sample means is normally distributed. However, parametric test can be misleading when this assumption is not satisfied. In this circumstance, non-parametric tests are the alternative methods available, because they do not required the normality assumption. Non-parametric tests are the statistical methods based on signs and ranks (Nahm, 2016).

Analysis of Variance (ANOVA) is a method for testing the equality of three or more population means, based on the analysis of sample variances. Some assumptions are necessary to use ANOVA: (a) Normally distributed populations, (b) Populations have the same variance (or the same standard deviation), (c) Samples are random and mutually independent. (d) The different samples are obtained from populations classified in only one category (Bennington & Thayne, 1994; H. D. O. G. da Silva et al., 2021; de Araújo et al., 2021; Sierra Murphy, 2024).

The Tukey test consists of comparing all possible pairs of means and is based on the minimum significant difference (M.S.D.), taking into account the group percentiles. When calculating the M.S.D., the distribution of the studied amplitude, the mean square of the ANOVA residuals and the sample size of the groups are also used (Batista et al., 2015).

3 MATERIALS AND METHODS

This work can be classified as applied research, as it aims to provide improvements in the current literature, with normative empirical objectives, aiming at the development of policies and strategies that improve the current situation (Bertrand & Fransoo, 2002). The approach to the problem is quantitative, as is the modeling and simulation research method. The research stages were carried out following the sequence shown in Figure 1.
3.1 CASE 1

The case shows a case study carried out in a company in the South Fluminense Region, in the state of Rio de Janeiro. The company is a metallurgical plant, and it is desired to compare the hardness of 4 types of Paints for Industrial use for painting equipment in the company. The assumption of Normality was checked and then Analysis of Variance and Tukey's Test were used.
Table 1

*Hardness according to each type of operator and each type of paint.*

<table>
<thead>
<tr>
<th>Paint</th>
<th>Dureza</th>
<th>Temp</th>
<th>Operador</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mistura 2</td>
<td>14.9</td>
<td>30.3</td>
<td>1</td>
</tr>
<tr>
<td>Mistura 3</td>
<td>13</td>
<td>30.9</td>
<td>2</td>
</tr>
<tr>
<td>Mistura 4</td>
<td>15</td>
<td>30.5</td>
<td>2</td>
</tr>
<tr>
<td>Mistura 1</td>
<td>17</td>
<td>29.4</td>
<td>3</td>
</tr>
<tr>
<td>Mistura 1</td>
<td>13.9</td>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>Mistura 3</td>
<td>16.4</td>
<td>29.6</td>
<td>1</td>
</tr>
<tr>
<td>Mistura 1</td>
<td>10.4</td>
<td>29.6</td>
<td>2</td>
</tr>
<tr>
<td>Mistura 2</td>
<td>3.2</td>
<td>29.9</td>
<td>2</td>
</tr>
<tr>
<td>Mistura 2</td>
<td>1.9</td>
<td>28.9</td>
<td>2</td>
</tr>
<tr>
<td>Mistura 2</td>
<td>7.3</td>
<td>28.6</td>
<td>1</td>
</tr>
<tr>
<td>Mistura 4</td>
<td>17.8</td>
<td>30.8</td>
<td>3</td>
</tr>
<tr>
<td>Mistura 2</td>
<td>9.6</td>
<td>30.8</td>
<td>3</td>
</tr>
<tr>
<td>Mistura 4</td>
<td>22.9</td>
<td>28.9</td>
<td>1</td>
</tr>
<tr>
<td>Mistura 4</td>
<td>17.4</td>
<td>30.6</td>
<td>3</td>
</tr>
<tr>
<td>Mistura 3</td>
<td>13.3</td>
<td>29.8</td>
<td>3</td>
</tr>
<tr>
<td>Mistura 1</td>
<td>19.3</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>Mistura 1</td>
<td>16.1</td>
<td>29.9</td>
<td>3</td>
</tr>
<tr>
<td>Mistura 3</td>
<td>6.5</td>
<td>30.6</td>
<td>2</td>
</tr>
<tr>
<td>Mistura 1</td>
<td>11.7</td>
<td>30.1</td>
<td>3</td>
</tr>
<tr>
<td>Mistura 4</td>
<td>16.9</td>
<td>30.7</td>
<td>3</td>
</tr>
<tr>
<td>Mistura 4</td>
<td>18.4</td>
<td>29.9</td>
<td>1</td>
</tr>
<tr>
<td>Mistura 3</td>
<td>11.9</td>
<td>29.5</td>
<td>2</td>
</tr>
<tr>
<td>Mistura 2</td>
<td>14.5</td>
<td>29.9</td>
<td>1</td>
</tr>
<tr>
<td>Mistura 3</td>
<td>16.8</td>
<td>29.9</td>
<td>3</td>
</tr>
</tbody>
</table>

Source: Authors (2024)

**4 RESULTS AND DISCUSSION**

**4.1 CASE 1**

First, the assumptions of Normality of Hardness and Temperature observed in Figures 2 and 3 were checked. To continue testing Normality, the Anderson-Darling test is performed, which consists of the following Hypotheses:

Ho: Data follow the Normal Distribution

H1: Data do not follow the Normal Distribution

The Anderson–Darling test is a statistical test of whether a given sample of data is drawn from a given probability distribution. In its basic form, the test assumes that there are no parameters to be estimated in the distribution being tested, in which case the test and its set of critical values is distribution-free. However, the test is most often used in contexts where a family of distributions is being tested, in which case the parameters of that family need to be estimated and account must be taken of this in adjusting either the test-statistic or its critical values. When applied to testing whether a normal distribution adequately describes a set of data,
it is one of the most powerful statistical tools for detecting most departures from normality. Anderson–Darling tests are available for testing whether several collections of observations can be modelled as coming from a single population, where the distribution function does not have to be specified. Anderson-Darling (AD) test is a modification of the Cramer-von Mises (CVM) test. It differs from the CVM test in such a way that it gives more weight to the tails of the distribution (Mohd Razali & Bee Wah, 2011).

If the p-Value is less than 0.05 (5%), Ho is rejected and H1 is accepted, otherwise Ho is accepted.

Figure 2 shows that the p-value is greater than 0.05 (5%), so Ho is accepted, indicating that the data is Normalized.

**Figure 2**

*Anderson-Darling Normality Test for Hardness*

![Anderson-Darling Normality Test for Hardness](image)

Note. Source: Authors, (2024)

Figure 3 shows that the data is normalized for Temperature.
Using Analysis of Variance and Tukey's Test to Compare Four Brands of Industrial Paint

Figure 3

*Anderson-Darling Normality Test for Temperature*

![Anderson-Darling Normality Test for Temperature](image)

Note. Source: Authors, (2024)

Once the Normality of the Data was guaranteed, the Analysis of Variance of the Data was carried out, which can be seen in Table-2. The null and alternative hypotheses to be tested in the Analysis of Variance are:

Ho: The population means are equal.

H1: The population means are different, i.e. at least one of the means is different from the others.

**Table 2**

*Analysis of Variance for Hardness*

<table>
<thead>
<tr>
<th></th>
<th>Fonte</th>
<th>GL</th>
<th>SQ (Aj.)</th>
<th>QM (Aj.)</th>
<th>Valor F</th>
<th>Valor-P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paint</td>
<td>3</td>
<td>281.7</td>
<td>93.90</td>
<td>6.02</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>Erro</td>
<td>20</td>
<td>312.1</td>
<td>15.60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23</td>
<td>593.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Source: Authors, (2024)

As the p-value is less than 0.05, Ho is rejected and H1 is accepted, i.e. at least one of the paints has a higher hardness than the others. Table 3 shows the Summary of the model and A low predicted $R^2$ value (24.32%) indicates that the model generates inaccurate predictions for new observations. The inaccuracy may be due to the small size of the groups. Therefore, the engineer should not use the model to make generalizations beyond the sample data.
Table 3

Model summary

<table>
<thead>
<tr>
<th>S</th>
<th>R²</th>
<th>R²(adj)</th>
<th>R²(pred)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.95012</td>
<td>47.44%</td>
<td>39.56%</td>
<td>24.32%</td>
</tr>
</tbody>
</table>

Note. Source: Authors, (2024)

The Minitab 19 software performs the Tukey test and the result can be seen in Figure 4. If an interval does not contain a zero, the corresponding averages are significantly different.

Figure 4

Tukey test for the 4 paints

All the confidence intervals for the remaining pairs of averages include zero, which indicates that the differences are not significant. The graph that includes Tukey's simultaneous confidence intervals shows that the confidence interval for the difference between the means of Mixtures 2 and 4 is 3.114 to 15.886. This range does not include zero, which indicates that the difference between these means is statistically significant. Paint 4 is therefore the one with the highest hardness and is statistically different from the others.
5 FINAL CONSIDERATIONS

In conclusion, Analysis of Variance is a very viable option when the assumptions of normality can be guaranteed in a data sample. If the analyzed population variable follows a normal distribution, a parametric test can be applied. When ANOVA proves that there is no equality between the means, Tukey's test clearly shows which sample is different from the others. In this specific case of the company, it can be concluded that Paint 4 is the best because it has the hardness that is statistically different from the others.

REFERENCIAS


