CASH FLOW AND ITS COMPONENTS IN INVESTMENT VALUATION

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ABSTRACT

Objective: The objective of this study is to provide a reflection on concepts and techniques used in investment analysis and evaluation with the aim of contributing to their clarification, as well as their practical application.

Theoretical Framework: The article begins with a brief review of the literature on the concept of cash flow, continuing with the suitability of the concept in investment analysis and a detailed analysis of its components from a theoretical and practical point of view, in the context of investment appraisal.

Method: The descriptive research method was used, as it is the most appropriate to the objective, making it possible to analyze the values relating to an investment project, as a way of demonstrating the validity and relevance of the concepts presented.

Results and Discussion: Details are given of how the periodic cash flow portions are constructed, how the value of the components is determined for the times after the time of analysis and their transformation into equivalent values that make it possible to transform the periodic values into the project's cash flow.

Conclusion: The concept of cash flow was discussed and clarified, emphasizing the temporal consideration of periodic non-addable cash flows. An example was given of how periodic cash flows can be determined beyond the study periods in concrete situations involving investments in real assets.

Keywords: Cash Flow, Residual Value, Working Capital, Periodic Cash Flow, Discount, Perpetuity.

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CASH FLOW E OS SEUS COMPONENTES EM AVALIAÇÃO DE INVESTIMENTOS

RESUMO

Objetivo: O objetivo deste estudo é providenciar uma reflexão sobre conceitos e técnicas, utilizados em análise e avaliação de investimentos com o objectivo de contribuir para a sua clarificação, bem como a sua aplicação prática.

Referencial Teórico: O artigo começa por uma breve revisão da literatura sobre o conceito cash Flow, prosseguindo na adequação do conceito em análise de investimentos e numa análise detalhada sobre as suas componentes do ponto de vista teórico e de prática, em contexto de avaliação de investimentos.

Método: Foi utilizado o método de pesquisa descritivo, dado ser o mais adequado ao objetivo, possibilitando analisar os valores relativos a um projeto de investimento, como forma de demonstrar a validade e relevância dos conceitos apresentados.

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INTRODUCTION

The decision to invest is one of the most important decisions a manager faces because it involves sacrificing resources in the immediate present with uncertainty about future benefits. Making a decision to invest requires multidisciplinary, engineering, strategic, financial, qualitative and quantitative analysis, prior to the phase known as economic and financial analysis of one or more investment alternatives to respond to a problem.

Economic-financial analysis reduces the heterogeneity of investments and their purposes to the study of a series of monetary flows generally called cash-flow. The concept is
used in most works on financial management and investment analysis and evaluation. However, not always with the same meaning. A reference will be made illustrating different views of the concept, without exhaustive concerns, but with emphasis on reference works in financial management, in general.

2 THEORETICAL REFERENCE

2.1 THE CONCEPT OF CASH FLOW IN INVESTMENT ANALYSIS

The concept of “cash flow” was the subject of great debate in the United States. American financial analysts, in the 60s of the last century, defined it as “net profit plus amortizations. It may or may not include provisions for deferred taxes as well as other charges not immediately payable” (Lauzel, 1977 p.107). This definition clearly highlights the still existing ties to the technical-accounting analyzes of the new theoretical contribution of a strictly financial nature. This is a definition associated with a calculation method, a practice still common in several textbooks, but not suitable for the financial matrix of the concept and therefore not suitable for investment analysis purposes. First of all because, in its formulation, it uses economic concepts, non-financial and disconnected from the temporality of its actual occurrence.

According to Menezes (1999), the use of the term “cash flow” appeared in the 19th century, XIX, but it was only in 1971 that it was considered mandatory for use by the American Institute of Stock Exchange Operations (Menezes, 1999 p.108). It defines it as a financial concept that covers the flows, and not just the balances, of the company's cash or treasury, occurring during a certain period, historical or future, well demarcated, and may be limited to exploration - Operational Cash Flow (CFO) or cover all financial flows. It reinforces the idea of a financial concept, although profitability (economic concept) influences, positively and negatively, the company's treasury.

Marques (1984, p.442) defines the concept as follows: "For us, cash-flow is what it is: the net flow of treasury. This is the original understanding of the expression that the Americans introduced and is also the most useful and clearer”.

With great similarity, Menezes and Marques present the concept of “cash flow” as the company’s financial flows. Treasury and cash or even financial flow are expressions that do not raise any ambiguity for the company's financial analysis. However, in investment analysis, we not only separate the monetary flows (treasury) related to investment from the monetary...
flows related to financing, but, firstly, we analyze the flows associated with investment and only, in a second stage, we analyze financing. On the other hand, in the analysis of investment decisions, the analysis focuses on flow estimates and not on actual flows that occurred.

In investment analysis, Brealey & Myers (1992) states that cash flows must be considered after taxes and when the respective receipts and payments occur, or are expected to occur. This is yet another aspect that refers to monetary flows and not to accounting concepts.

From this perspective, in investment analysis, total cash flow involves investment flows (cash outflows), operating flows (cash inflows and outflows) and the variation flows of current assets associated with the investment cycle. Operations, generally referred to as Working capital or working capital needs.

(Ross et al., 2010) considers the concept “cash flow” to be probably one of the most important pieces of information that can be obtained from financial statements, defining it as the difference between the number of dollars coming in and the number of dollars going out in the business in a given year. Adding that the financial statements do not present this information as would be desired. It presents what it calls Cash Flow’s identity:

\[
\text{Cash flow from assets} = \text{Cash flow to creditors} + \text{Cash flow to shareholders}.
\]

Cash Flow from the perspective of interested entities according to Menezes (1999, p.109). But it is precisely the notion of cash flow from assets that appears to be most relevant to the study of investments. This identity considers the cash flow generated after paying taxes. In other words, it deals with the cash flow that is free to remunerate the invested capital, regardless of its origin.

The cash flow of assets involves three components: Capital expenditures (I), Operational Cash Flow (CFO) and net variation in working capital (or variation in working capital requirements-NFM) (∆NFM):

- Capital Expenditures (I) refer to expenses on fixed assets (purchases minus sales);
- Operational Cash flow (CFO) is the monetary flow from daily production and sales activities that does not include any financing expenses as it is not considered an operational expense;
- Variation (∆) in (NFM) corresponds to the net variations in assets involved in the current cyclical business.

In the context of investment analysis, it is not considered appropriate to use concepts of results because they are obtained using accounting criteria with a time lag between their construction and the corresponding monetary flow and because there may be different practices in the construction of these indicators. The monetary flow does not have two interpretations.
and its quantification takes into account the periods in which it occurs, an aspect of enormous importance in economic-financial analysis and generally ignored from an accounting perspective.

According to the authors (Ross et al., 2010), Cash Flow from assets is sometimes called “free” cash flow”, meaning that it is free cash flow (after taxes) to be distributed to creditors and shareholders.

Regardless of the etymological issue, the concept of cash flow of assets, according to Ross et al. (2010), is the one that we consider suitable for investment analysis as it highlights the three dimensions of Cash Flow that reflect operational performance, the effect of long-term management decisions with regard to investment decisions and the effect of financial/operational decisions on how to finance working capital needs (NFM).

The “cash flow” concept, for analyzing an investment project, is then the measure of all monetary flows relating to investment expenditure, in capital and in variation in NFM and all operating income and expenses, over a series of periods, over time. This is the concept for analyzing an investment, regardless of, or before, considering any form of financing.

To further clarify the concept, it is important to analyze how cash flow is calculated in each of the analysis periods and understand that the cash flow of the investment project is the result of periodic cash flows, after being transformed into equivalent units.

2.2 TIME IN INVESTMENTS

The evaluation, a priori (ex-ante) of investments, is made on a financial basis on the estimated future flows of expenses and income, at the time in which they occur, and over a broad time horizon of the economic life of the assets that make up the investment. This horizon comprises the implementation time and the exploration time or delivery of benefits to their recipients.

The timeline has the before and after, of the start of the investment execution, as shown in figure 1.
The expected date for the start of construction or installation of the investment project is the reference date for the processing of data that we will use in the economic-financial analysis of an investment in real assets. This is the moment, Moment zero and not period zero (red arrow) to which the cash flow analysis that serves as the basis for the economic-financial assessment of the investment must be reported, regardless of the moment in which the analysis is being carried out.

The time before the start of project implementation is sometimes as long or longer than the economic life of the project. The issues addressed there are of enormous importance for the success or failure of the project, an aspect abundantly covered in the literature.

The implementation, that is, the investment in real assets, begins for a period, of varying length, with the construction or implementation of these assets. The operation follows, which sometimes begins, albeit partially, before the end of full deployment and other times only after the full deployment of assets and training of operational employees.

The operation time deserves particular attention. It can be said that operating time is directly related to the economic life of the goods used, but also to several other factors. It is always finite without prejudice to the possibility of making investments to extend the normal lifespan of installed physical assets.

The time for preparing monetary quantitative data for analysis and evaluation of an investment in real assets is generally less than the possible operating time (lifetime) for these real assets. The time for analysis generally comprises the investment time plus 5 to 10 years of operation, as shown in figure 2.
The question then arises of how to incorporate monetary flows that occur beyond the analysis time into the analysis? We will seek to answer this question in each installment of the flows.

2.3 CASH FLOW QUANTIFICATION FOR ANALYSIS

When quantifying the cash flow of an investment, we will follow the previously presented definition of three installments, now introducing details of the components and aspects related to time that are considered in the investment analysis.

2.3.1 Cash Flow From Investment Expenses

The quantification of this portion of cash flow must be exhaustive and supported by evidence: budgets or technical quantifications with price. It consists of quantifying expenditure on construction or acquisition of fixed assets and implementation of the productive capabilities of the investment project. The cash flow will be a cash flow of disbursements, cash out, and therefore we represent it with a sign (-Y) during all periods of construction or implementation. The expenses relate to all construction and implementation actions of the project and include design expenses, analysis of alternatives, land, equipment, projects, and other expenses necessary to make the projected investment operational.

Investment cash flow must also include the financial flow arising from opportunity costs, an issue that often arises when expanding existing projects. The assessment must be made from a “with or without” project perspective and not from a “before and after” perspective, as mentioned (Brealey & Myers, 1992, p-99).
In studies whose analysis time goes beyond the normal lifespan of the assets, investment expenses are considered to include expenses with replacing the assets' productive capabilities, or acquiring new ones. Let us designate them, for representation purposes, as \((Y_m)\). Sometimes, for simplicity, the value corresponding to the value of amortizations is taken for the size of these expenses.

Another flow that needs to be considered in an investment analysis is the expense or income from the sale or decommissioning of end-of-life assets, commonly known as residual value. This flow may be input (+\(Y_r\)) or output (-\(Y_r\)).

In a synthetic way, we represent the investment cash flows, in each period of time, as illustrated in figure 3:

**Figure 3**

*Investment cash flow*

\[-Y_1; \ -Y_2; \ -Y_3; \ -\ldots; \ -\ldots; \ -Y_m; \ \ldots; \ \ldots; \ -Y_rou + Y_r\]

### 2.3.2 Operational Cash Flow or operational Cash Flow

The quantification of operational cash flow corresponds to inflows (sales) minus outflows (excluding financial charges), that is, it is equal to Sales – expenses (which express money outflow) – Taxes on earnings.

CFO quantification can be done through: Earnings before interest and taxes (EBIT) + depreciation – taxes. However, this way of measuring is not pedagogically appropriate as it involves non-monetary variables. This pedagogical aspect is important. Mathematics is extremely important to provide a basis, however it is only appropriate to apply mathematical models if a causal relationship between the variables used is established.

Another aspect that Ross draws attention to is that the method of calculating operational cash flow, stated here, differs from the method of calculation used by those who generally analyze financial statements. To explain the difference, the example shown in figure 4 is constructed. Starting from an income statement, two ways of calculating cash flow are presented (in the center) from the perspective of Ross, et al (2010) and (right side ) the calculation method that is generally used in financial statements.
The calculation method that is defended and that is most suitable for investment analysis is the first (in the center). It is important to note that the tax rate is applied to EBIT, because an investment analysis is primarily carried out regardless of the financing method. When calculating using the standard accounting method, the tax rate is the same, but is applied to the result after taxes.

Quantifying cash flow in future terms proves to be extremely difficult. Obtaining data about the future, whether in expenses or income, is subject to many uncertainties and therefore, being difficult to obtain, information is generally only constructed for a short period of the operation, basically until it reaches operational stability. It is therefore assumed that the analysis time period corresponds to the time necessary for the project operation to achieve stability in its functioning and in the generation of cash flow. Depending on the specific characteristics of the real assets, the quantification of operational cash flow can then be done for a period between five and ten years (in infrastructure projects longer periods are used, 20 and 30 years). For the remaining period, that is, the period beyond the time in which the details of the flows are made, which can be assumed to be unlimited in length, an estimate is made using mathematical processes, based on duly substantiated and explained assumptions. It is important to say that, depending on the size of the update rate used, the monetary flows of the future duration, despite being said to be “unlimited”, are quickly extinguished. To calculate this portion, a perpetuity of the cash flow from the period following the last detailed year is taken:

\[
\frac{\text{Cash Flow}}{r} 
\]

where:

\[
\text{Cash Flow} = \text{Vendas e serviços prestados} - \text{Gastos operacionais} - \text{Impostos (25%)/EBIT}
\]

\[
\text{EBIT} = \text{Vendas e serviços prestados} - \text{Gastos operacionais} - \text{Impostos (25%)/EBIT}
\]

\[
\text{CFO} = \text{EBIT} - \text{Amortizações} - \text{Juros soportados} - \text{Impostos (taxa 25%)}
\]

\[
\text{Rendimento Líquido (Net Income)} = \text{CFO}
\]
r represents the opportunity cost of capital.

Or it is assumed that cash flow grows at a constant rate (g) and, in this case, its value will be estimated using a Gordon model-type expression (see Brealey, R. A., Myers, S. & Allen, F. (1992).

\[
\text{Cash flow}_{t+1} = \frac{\text{Cash flow}_t \times (1+g)}{r-g}
\]

(2)

where:

- r represents the opportunity cost of capital and g represents a constant rate of growth.

Operational cash flow corresponds to the monetary flows of income and operational expenses, resulting from sales and expenses with raw materials, FSE, personnel and other expenses related to the operation, translated into receipts and payments, free of direct taxes as if demonstrated previously (see figure 4) \([R-(G+I)]\) .

In the first years of operation, cash flow may be cash out, as the project may not yet have a volume of activity above the critical point. Critical point corresponds to the situation in which the value of sales reaches, or equals, the total costs. It is a situation of zero operational results. Only with sales greater than total costs will a positive operational result be obtained.
Figure 5

*Periodic calculation of operational cash flow*

<table>
<thead>
<tr>
<th>Periodos anuais</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>5+1</th>
<th>5+2</th>
<th>…</th>
<th>5+n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluxo de recebimentos</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluxo de Pagamentos</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluxo de Impostos*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash flow periódico</td>
<td>+Σ</td>
<td>+Σ</td>
<td>+Σ</td>
<td>+Σ</td>
<td>+Σ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Period 1 corresponds to the first period of operation.
Taxes are calculated as shown in figure 4.

Often, textbooks present the calculation of operational cash flow based on the income statement and, starting from the net result, add items that do not correspond to monetary flow, and others that do not relate to the operation, and that had deducted to calculate the net result. This method of calculation, although it can produce, in the absence of interest, the same magnitude as the aforementioned method, we do not consider it pedagogically appropriate as it does not associate the method of calculation with the cause that generates the flows.

### 2.3.3 Cash Flow to “Working Capital” or NFM

In an investment in real assets, put into operation, it is necessary to consider the working capital needs (See Neves, Ross, and others) arising from buying and selling on credit and the need to hold inventories in the operation in question.

An investment, once in operation, requires financial resources, on a cyclical basis, to buy and sell on credit and to hold inventories. The size of Working capital depends on the size of the operation, inventory rotation times, credits granted and obtained. These resources will be required throughout the operating time. Its amount is not cumulative, it corresponds to variations in “working capital” and may correspond, in each period, to cash out or cash in monetary flows, depending on whether it is necessary to invest in NFM or reduce, disinvest, the amounts involved in NFM.

Working Capital (figure 6) integrates the cyclical needs constituted by the value of inventories; by the value of credits received (value of sales that are permanently to be received from customers) and the cyclical resources made up of credit obtained, permanently, from
suppliers and even from the State (by the difference between input VAT and input VAT). Sometimes a reserve amount of money is also included in NFM to deal with unforeseen situations.

At the end of the project's life, the amounts receivable and payable and the inventories are extinguished. In the end, therefore, a disinvestment occurs as there are no longer any cyclical needs, thus obtaining, on that date, a flow, normally, of cash in, in the amount involved in Working capital. Some businesses have supplier credit that exceeds inventory and customer credit needs and in these circumstances working capital represents a cash in and its resolution a cash out. It is assumed, for simplicity, that all credits and all debts are received and paid and all inventories are sold.

**Figure 6**

*Cash Flow to NFM or Working capital*

\[-WK_1; - \Delta WK_2; - \Delta WK_3; \ldots; + WK_n\]

Note: Period 1 corresponds to the first period of operation

Some authors deduct “working capital” from the cash flow generated in the operation. However, these portions of cash flow are separated because working capital is related to management decisions (financial and management policies) and not operational performance. It is considered important that the operational cash flow is known in its entirety, because it is in the operation that the benefits of the investment are generated, translated into monetary flow, and therefore they must be clearly explained.

**2.3.4 Periodic Cash Flow of the set of Installments (A+B+C) of Investment Cash Flow**

Figure 7, with all installments of the periodic cash flow for an investment project, with an analysis time of 5 years of operation, can then be presented:
**Figure 7**

*Periodic cash flow for a defined time*

<table>
<thead>
<tr>
<th>Periodos anuais</th>
<th>Período de operação</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
</table>

It is assumed that monetary flows occur at the end of each period. Time 6 appears here to represent what goes beyond the 5 periods considered in detail, as an example. We will see in the numerical example how to bring these values to period 5 of the operation.

### 3 METHODOLOGY

This study uses the descriptive method in terms of its objective, going through a literature review on the topic addressed. Its purpose is to provide a reflection on concepts and techniques used in the analysis and evaluation of investments with the aim of contributing to their clarification.

The descriptive research method was used, as it is the most appropriate for the objective, making it possible to analyze the values related to an investment project, as a way of demonstrating the validity and relevance of the concepts presented in the theoretical framework, in the evaluation of investment projects. The objective is to build an example that can be applied empirically.

### 4 RESULTS

For the purposes of demonstrating investment analysis, fictitious data is presented below, relating to the cash flows of an investment project, with an analysis period of 8 years, whose values are shown in figure 8.
Figure 8

*Flows of an investment project in real assets*

<table>
<thead>
<tr>
<th>Descrição</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Gastos de investimento</td>
<td>-150</td>
<td>-300</td>
<td>-100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B Cash Flow de Operação [R-(G+I)]</td>
<td>150</td>
<td>250</td>
<td>280</td>
<td>300</td>
<td>306</td>
<td>312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C Necessidades cíclicas em &quot;WK&quot;</td>
<td></td>
<td>-15</td>
<td>-10</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>Cash flow periódico</td>
<td>-150</td>
<td>-300</td>
<td>35</td>
<td>240</td>
<td>277</td>
<td>298</td>
<td>305</td>
<td>311</td>
</tr>
</tbody>
</table>

Note: Annual periods. Values in monetary units, at constant prices.

The cash flow of each annual period corresponds to the sum (column) of the three monetary flow components in each period. Observe the sign that informs whether we are experiencing cash outflow or cash inflow.

Note that, in the example, the assets are only complete in time 3, which corresponds to period 1 of the operation. At the end of its economic life, its residual value may translate into a benefit (cash in) or an expense (cash out). Let us admit that its residual value, at the end of a life cycle, will be an uninstallation expense of 50 monetary units (u.m.). Based on this information, how could we evaluate the underlying investment?

We have to quantify the cash flow that may occur beyond the 5th year of operation and also know whether, at the end of the assets' life, there is any flow. To do this, we will have to formulate several hypotheses, particularly regarding the life cycle of the assets. In this example we will consider that the asset life cycle is 10 years.

Two calculation hypotheses are presented:

- **H1**) One in which assets have a life cycle (duration) of 10 years and
- **H2**) Another in which assets have different life cycles (duration).

Note that by assuming limited time and unlimited time, we are assuming that they are different projects. The use of the same known real data highlights the importance of assumptions and hypotheses in project evaluation.

4.1 INVESTMENT ANALYSIS IN HYPOTHESIS H1

If the assets have an estimated duration of 10 years, and at the end of that time their deactivation involves an outflow of 50 (one), we must consider this flow (residual value) in the 5th year of operation (time of this study). The conversion of the flow from the 10th to the 5th year is done through updating or discounting. To do this, it is necessary to consider a discount
rate. Be it, for example, 9%, the discount rate Discount rate corresponds to the opportunity cost of capital.

**Figure 9**
Transformation of a.m. from a future time into a present time

<table>
<thead>
<tr>
<th>Nº de períodos posteriores ao tempo de operação</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Periodos de operação</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Factor de transformação Com r=9%:</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor de transformação Com r=9%:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Valor equivalente no fim do ano 5</td>
<td>-32,50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The residual value of the assets, to be considered in the periodic cash flow, in year 5 of operation, in this example, will be -32.5 (u.m.), a disbursement.

Given that the study period is 5 years, but we know that the assets will continue their activity in subsequent years, until the end of their life cycle, we have to estimate the operational cash flow that will be generated in this remaining period. We know that operating cash flow is stabilized in the 5th year of operation, growing at 2% per year. Then, we will be able to estimate the cash flow after the fifth year, until the end of the useful life of the installed assets, 10 years in this case. Assuming constant growth, at a rate of 2% per year, the remaining cash flows are estimated and then the flows for each period are updated to the end of the 5th period. Once updated, they can be added to obtain 1259 (one) as shown in the following illustration.

**Figure 10**
Operating cash flow for periods after the 5th year, present value of future values

<table>
<thead>
<tr>
<th>Tempo de operação</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow de Operação [R-(G+I)]</td>
<td>312</td>
<td>318</td>
<td>325</td>
<td>331</td>
<td>338</td>
</tr>
<tr>
<td>Factor de transformação Com r=9%:</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor de transformação Com r=9%:</td>
<td></td>
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</tr>
<tr>
<td>Valor equivalente do cash flow de operação nos períodos 6 a 10, no final do ano 5</td>
<td>286</td>
<td>268</td>
<td>251</td>
<td>235</td>
<td>219</td>
</tr>
<tr>
<td>Valor equivalente da soma dos cash flow 6 a 10 no final do ano 5</td>
<td>1259</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The evaluation covers a period of 5 years, it can be assumed that the accumulated value of the investment in working capital could be extinguished which, in this case, would correspond to an inflow of 31 (one).

**Figure 11**

*Residual value of Working capital*

<table>
<thead>
<tr>
<th>Tempo desde o início do investimento</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tempo de operação</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Necessidades cíclicas em &quot;WK&quot;</td>
<td>-15</td>
<td>-10</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reembolso do &quot;WK&quot; no fim do tempo de estudo</td>
<td>31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At the end of period 5, the nominal value of the accumulated working capital is received (cash flow with a + sign).

Total periodic cash flow of H1: It is now possible to present, in full, the periodic cash flows of an investment in real assets for a life cycle, as shown in figure 12.

**Figure 12**

*Periodic cash flow of a project, an asset life cycle*

<table>
<thead>
<tr>
<th>Descrição</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Gastos de investimento</td>
<td>-150</td>
<td>-300</td>
<td>-100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gastos de desativação</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B <em>Cash Flow de Operação (R-(G+I))</em></td>
<td>150</td>
<td>250</td>
<td>280</td>
<td>300</td>
<td>306,0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash Flow de Operação (R-(G+I))de 6 a 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C Necessidades cíclicas em &quot;WK&quot;</td>
<td>-15</td>
<td>-10</td>
<td>-3</td>
<td>-2</td>
<td>-1,0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Desinvestimento de &quot;WK&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash flow periódico</td>
<td>-150</td>
<td>-300</td>
<td>35</td>
<td>240</td>
<td>277</td>
<td>298</td>
<td>1562,3</td>
</tr>
</tbody>
</table>

**4.2 INVESTMENT ANALYSIS IN HYPOTHESIS H2**

In long periods of asset life, its residual value or its deinstallation, it is assumed that it is, at present, tending to be null due to the effect of the update.

Regarding the estimation of future operational cash flow, after the 5th year of operation, we must immediately consider a change in the determination of operational cash flow. We continue to assume that the average normal life cycle of assets is 10 years. Therefore, in order for assets to be active in later life cycles, we will have to assume that we make additional investment in all new asset life cycles. One hypothesis, generally considered in evaluations, is to consider an amount of reinvestment expenses equivalent to the value of amortizations in every subsequent year. In this circumstance, the reinvestment value corresponds to the
amortization value. Amortizations occur annually, but reinvestments occur over asset life cycles.

For this reason, in the practice of evaluating investments, for different life cycles of assets, a cash flow reduced by the amount of amortizations is used for operational cash flow.

**Figure 13**

*Operating cash flow, for long periods, over the life of assets*

Admitting several life cycles of assets, the analysis would then have to be carried out with an operational cash flow reduced by the value of asset amortizations (see, for example, Pinto et al. (2023). The estimate of Perpetuity increasing at a constant rate (g), \([\text{Cash flow}]_{\text{6} \times (1+g)}/(\text{r-g})\), would now be made based on a lower cash flow. The estimate of the value of the operational cash flow for the period after that. 5th year of operation is carried out, as already explained and, for the example presented, illustrated below.

**Figure 14**

*Future tense operational cash flow*

Total periodic cash flow of H2: It is now possible to present, in full, the periodic cash flows of an investment in real assets for an unlimited life cycle, as shown in figure 15.

**Figure 15**

*Periodic project cash flow, long asset life cycles*
A reading of the previous figures allows us to reflect on the ways in which periodic cash flow is calculated depending on whether we are considering an unlimited lifetime and a limited lifetime, within the framework of other assumptions made, for investments in real assets.

4.3 FROM PERIODIC CASH FLOW TO PROJECT CASH FLOW

The need then arises to determine the project’s cash flow. Note that periodic cash flows cannot be added! To be addable, future periodic cash flows must be transformed into equivalent values of the present, with the present being the moment in which the implementation of investment in real assets begins.

The sum of the value of periodic cash flows, after being transformed into present values, or updated, corresponds to the cash flow of the investment.

The following two figures, 16 and 17, present the cash flow calculations for the example given, depending on the useful life of the assets under analysis.

**Figure 16**

*Project Cash Flow, an asset life cycle*

<table>
<thead>
<tr>
<th>Início do período 1 = momento zero</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow periódico para todo o tempo de vida do projecto</td>
<td>1</td>
<td>-150</td>
<td>-300</td>
<td>35</td>
<td>240</td>
<td>277</td>
<td>298</td>
</tr>
<tr>
<td>Factor de transformação Com r=9%:</td>
<td>(1 \over (1 + r)^n)</td>
<td>0.91743</td>
<td>0.84168</td>
<td>0.772183</td>
<td>0.708425</td>
<td>0.649931</td>
<td>0.596267</td>
</tr>
<tr>
<td>Cash Flow descontado</td>
<td>-138</td>
<td>-253</td>
<td>27</td>
<td>170</td>
<td>180</td>
<td>178</td>
<td>855</td>
</tr>
<tr>
<td>Cash Flow do Projecto</td>
<td>1019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 17**

*Project Cash Flow, various asset life cycles*

<table>
<thead>
<tr>
<th>Início do período 1 = momento zero</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow periódico para todo o tempo de vida do projecto</td>
<td>1</td>
<td>-150</td>
<td>-300</td>
<td>-20</td>
<td>185</td>
<td>222</td>
<td>243</td>
</tr>
<tr>
<td>Factor de transformação Com r=9%:</td>
<td>(1 \over (1 + r)^n)</td>
<td>0.91743</td>
<td>0.84168</td>
<td>0.772183</td>
<td>0.708425</td>
<td>0.649931</td>
<td>0.596267</td>
</tr>
<tr>
<td>Cash Flow descontado</td>
<td>-138</td>
<td>-253</td>
<td>15</td>
<td>131</td>
<td>144</td>
<td>145</td>
<td>2318</td>
</tr>
<tr>
<td>Cash Flow do Projecto</td>
<td>2333</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
C_1 (1 + r)^1 + C_2 (1 + r)^2 + C_3 (1 + r)^3 + \ldots + C_n (1 + r)^n = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}
\]
5 CONCLUSION

In conclusion, the concept of “Cash flow” in general and its particularities for the analysis of investments before, or independently, of the financing method, was discussed and clarified. The temporal consideration of non-addable periodic Cash Flow was emphasized. It was exemplified how periodic Cash flow can be determined beyond the study periods in concrete situations of investments in real assets. It was illustrated how periodic cash flow should be transformed into investment project cash flow.

As a future investigation, the association of the project's cash flow with a profitability indicator and validation criterion for decision-making on investments could be considered.

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REFERENCES


