ABSTRACT

Theoretical Framework: The teaching of calculus is indispensable in many scientific careers, and maxima and minima problems hold a prominent place among its contents. However, students often encounter difficulties solving optimization problems.

Objective: Considering this, our study aims to determine the perception of calculus teachers regarding the importance of contextualization and the use of diverse resources when they teach these problems.

Method: Our study has a descriptive nature and to achieve our goal, a sample of university calculus professors from the Dominican Republic was analyzed using a Likert-type test.

Results and Discussion: The results demonstrate a favorable trend towards contextualization and the integration of resources in teaching maxima and minima problems, although some professors still prefer traditional methods based on textbooks.

Research Implications: The findings highlight the importance of exploring contextual teaching strategies, promoting resource diversity, and adopting innovative pedagogical approaches to enhance learning, comprehension, and student motivation.

Keywords: Calculus, Mathematical Education, Maxima And Minima, Optimization Problems, Problem Solving, University.
Método: Para isso, foi realizado um estudo descritivo, analisando uma amostra de professores universitários de cálculo na República Dominicana, aplicando uma pesquisa do tipo Likert.

Resultados e Discussão: Os resultados mostram uma tendência favorável à contextualização e integração de recursos no ensino de problemas de máximos e mínimos, embora ainda sejam identificados professores que preferem seguir métodos tradicionais baseados em livros didáticos.

Implicações da Pesquisa: Os resultados destacam a importância de explorar estratégias de ensino contextuais, promover a diversidade de recursos e adotar abordagens pedagógicas inovadoras para aprimorar o aprendizado e melhorar a compreensão e a motivação dos alunos.

Palavras-chave: Cálculo, Educación Matemática, Máximos y Mínimos, Problemas de Optimización, Solución de Problemas, Universidad.

CONCEPCIONES DEL PROFESORADO SOBRE LA ENSEÑANZA DE LA RESOLUCIÓN DE PROBLEMAS DE MÁXIMOS Y MÍNIMOS A ESTUDIANTES DE CÁLCULO: UN ENFOQUE EN LA EDUCACIÓN UNIVERSITARIA

RESUMEN

Marco Teórico: La enseñanza del cálculo es indispensable en muchas carreras científicas y los problemas de máximos y mínimos ocupan un lugar destacado entre sus contenidos. Sin embargo, en ocasiones el alumnado presenta dificultades al resolver problemas de optimización.

Objetivo: Teniendo esto en cuenta, nuestro estudio tiene como objetivo conocer cuál es la percepción de docentes de cálculo sobre la importancia de la contextualización y la utilización de recursos variados en la enseñanza de estos problemas.

Método: Para ello, se realizó un estudio descriptivo, analizando una muestra de profesores universitarios de cálculo de la República Dominicana, aplicando una encuesta tipo Likert.

Resultados y Discusión: Los resultados muestran una tendencia favorable hacia la contextualización y la integración de recursos en la enseñanza de problemas de máximos y mínimos, aunque se identifican aún profesores que prefieren seguir métodos tradicionales basados en libros de texto.

Implicaciones de la investigación: Los hallazgos subrayan la importancia de explorar estrategias de enseñanza contextual, promover la diversidad de recursos y adoptar enfoques pedagógicos inovadores para mejorar el aprendizaje y potenciar la comprensión y la motivación de los estudiantes.

Palabras clave: Cálculo, Educación Matemática, Máximos y Mínimos, Problemas de Optimización, Resolución De Problemas, Universidad.

RGSA adota a Licença de Atribuição CC BY do Creative Commons (https://creativecommons.org/licenses/by/4.0/).

1 INTRODUCTION

In recent years, international studies such as the PISA report have analyzed the mathematical competence of students, showing large differences between countries (Ministry of Education, vocational training and sports, 2023, p. 16). This makes it important to know various aspects related to the teaching and learning processes of mathematics at all educational
levels, with the aim of incorporating them into teaching as proposed by Yerizon et al. (2024), assessing the development of mathematical competence by students, as indicated by Torres et al. (2024) or Vega-Castro (2022).

Mathematical competence is assumed to be “having knowledge of, understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role” (Niss and Højgaard, 2011, p. 49).

In this sense, when we talk about mathematical competence we see that the connection of mathematics with real contexts is also highlighted:

The ability to reason mathematically and to formulate, employ and interpret mathematics to solve problems in a variety of real-life contexts. This includes concepts, procedures, data and tools to describe, explain and predict phenomena. This competency helps people understand the role that mathematics plays in the world and to exercise the judgments and make the informed decisions that thoughtful, constructive and engaged citizens of the 21st century need. (Ministry of Education, vocational training and sports, 2023, p. 16)

There is evidence in the scientific literature that some students face challenges when trying to assimilate the concepts of calculus due to its highly abstract nature and the requirement for a high level of conceptual understanding (Parameswan, 2007).

According to Orton (1983), most of the errors made when solving differential calculus problems are related to conceptual aspects. In Bezuidenhout’s (2001) study, it is stated that some of the difficulties and misconceptions in calculus expressed by students were the result of a teaching approach that emphasizes the procedural aspects of calculus and neglects a solid foundation of its fundamentals.

The findings of Guzmán and Vallejo (2004) show an incorrect interpretation of the concepts of maximums and minimums by using the second derivative criterion without carefully evaluating the solution obtained and by not taking into account that the domain of the function is closed, which is crucial when determining the maxima and minima of a continuous, bounded and differentiable function in its domain. Some researchers have tried to propose various strategies to improve the understanding of calculation concepts, among them is the option of incorporating technological resources. For example, Tall (1986) planned a cognitive approach to computation using gradient mapping and drawing programs in function representations.

Malaspina (2007) considers that university students who have already studied maximums and minimums in differential calculus subjects, when solving optimization problems, use the criteria of the first and second derivative almost mechanically, without
showing a scientific attitude that combines intuition, conjecture, formalization and rigor in the face of other problems in which the difficulty is not obtaining the optimal value of a continuous function defined in a closed and bounded interval.

According to Sánchez-Matamoros (2004) and Sánchez-Matamoros et al. (2006), some of the university students who have taken a mathematical analysis course present difficulties in understanding the concept of derivative and have difficulties establishing relationships between mathematical elements. Font (2000) indicates that it is important to design focused teaching so that students can coordinate the different representation systems. Font (2000) assumes that the calculation of $f'(x)$ from $f(x)$ is a process that is considered in three subprocesses, in which different modes of representation are involved:

- Ostensive ways of representing $f(x)$.
- The transition from an ostensive way of representing $f(x)$ to a way of $f'(x)$.
- Translations between the different ostensive forms of representation $f'(x)$.

Where ostensive means the form of the function that the teacher writes on the blackboard and the student observes on it. (p.282)

Brijlall and Ndlovu (2013) state, teachers must be aware of students' learning conflicts to reinforce the new concepts that are presented to them.

In the scientific training that every university student of mathematical analysis receives, the resolution of maximum and minimum problems is essential for various reasons, among others because it has practical applications, helps to understand fundamental concepts, develops analytical skills and prepares students for future studies and careers in various areas.

According to White and Mitchelmore (1996), in the field of calculus, the context in which an application problem is posed can vary between a genuine situation in the real world, an artificial situation inspired by reality, or an abstract mathematical context with a level of abstraction lower than the calculation concept that is going to be used.

Solving maximum and minimum problems in calculus requires an in-depth understanding of concepts such as the derivative, critical points, the concavity of a function, among others. These concepts are fundamental in mathematics and have extensive applications. For example, many problems in science, engineering, economics, and other disciplines can be formulated as maximization or minimization problems.

It could be said that the theory of maximums and minimums is concerned with the determination of optimal situations, that is, choices for which the functions have their maximum or minimum values.

Problems that seek to maximize or minimize a numerical function of a number of
variables, with these variables subject to certain constraints, form a general class that can be called optimization problems (Whitfield, 1967). Its relevance lies in its applicability to practical issues in science, economics, industry, etc. Therefore, it has become imperative to develop innovative approaches and alternative methods to address them effectively.

However, not all students interpret these types of problems adequately, and among the reasons for this, we can find that the teachers have not changed their teaching methods. Thus, the teaching methods used by the vast majority of teachers probably worked well in the past, but are not working well today; This makes it necessary to change educational strategies and the starting point is in the classroom (Crawford, 2004; Galindo & Frank de Lozada, 2023).

Sánchez-Matamoros et al. (2008) state that there is a widespread opinion that the planning of curricular decisions in educational policy, as well as the instructional actions carried out by teachers, should be based, among other aspects, on the findings derived from research. This implies the incorporation of alternative teaching methods for teaching mathematics. In such a way that a teacher's understanding of a new method will be demonstrated not only in his ability to reflect and identify its characteristics, but also in his ability to intervene and make decisions in relation to said method (Mometti, 2023).

One of the alternative methodologies is the one proposed by Crawford (2004) who describes strategies to help students interpret problems appropriately; called contextual teaching strategies. The words that identify these strategies are:

- **Relationship**: It means learning in the context of life experience.
- **Experimenting**: It consists of learning in the context of exploration, discovery and invention, that is, learning by doing.
- **Application**: It consists of learning concepts in the context of their implementation.
- **Cooperation**: It consists of learning in the context of sharing and interacting.
- **Transfer**: It consists of learning in the context of applying knowledge in new contexts or in new situations (not addressed in class).

These strategies that help develop understanding are called contextual teaching strategies and are based on how people learn to understand and how teachers teach to understand.

These strategies are focused on teaching and learning in context, a basic principle in constructivism, and different research suggests that they are strategies through which students can learn better (Díaz & Careaga, 2021; Inzunza-Cazare & Rocha Ruíz, 2024).

On the other hand, in recent years, university students in the Calculus 1 subject at the Technological University of Santiago (UTESA), Máximo Gómez campus, present many
difficulties in solving Maximum and Minimum Problems (extreme of a function), evidencing that this content is not assimilated correctly by them, as shown in the different tests and also in the opinions of the students.

Therefore, the objective of this research is to evaluate the practices and beliefs of teachers of this subject in relation to teaching maximum and minimum problems in the classroom, in order to identify areas of strength and opportunities for improvement in integration of resources, contextualization of problems and pedagogical approaches. The study seeks to understand teachers’ perception of the importance of contextualization and the use of resources in teaching optimization problems, as well as its impact on students' understanding and learning.

2 METHODOLOGY

This research will be descriptive, considering describing the phenomenon under study, providing reality data based on statistics. Our method will be analysis and synthesis, with the objective of studying the information that will support our research in the presentation and analysis of the results.

The sample population and indicators are 23 professors from the Technological University of Santiago – UTESA of Santo Domingo, who teach or have taught the subject of Calculus 1, 19 of whom are men and 4 are women. 73.9% of teachers have more than 10 years of teaching experience.

Statistical analyzes of qualitative values were performed with SPSS version 25 software. As an instrument, a Likert scale test of 10 questions related to aspects of teaching maximums and minimums in calculus courses was applied (Table 1). The scale was scored from 1 to 5 as follows: Strongly disagree: 1; Disagree: 2; Neutral, neither agree nor disagree: 3; Agree: 4; Totally agree: 5.

Table 1

<table>
<thead>
<tr>
<th>Applied instrument</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. When I explain solving maxima and minima problems, I explain the topic first and then solve problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. When I explain the resolution of maximum and minimum problems, I begin the topic with a problem to which an answer is given as the topic progresses.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. The problems I solve with my students are not related to daily life or a context.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. I usually do not create contextualized maximum and minimum problems to solve in the classroom.

5. I am convinced that students understand optimization problems better when they work on problems contextualized to their environment.

6. I mainly follow a textbook to teach my classes on maximum and minimum problems.

7. I feel more confident using a textbook to explain maximum and minimum problems.

8. The textbook that I use in the classroom does not propose maximum and minimum problems that are related to daily life or a context.

9. I do not consider it negative that students use the calculator to solve maximum and minimum problems.

3 RESULTS AND DISCUSSIONS

The analysis of the responses to question 1 reveals that 82.6% of teachers opt for a classic approach in which the topic is explained first and then maximum and minimum problems are solved (Figure 1). This indicates a dominant trend in the teaching practice of these teachers, probably opting for this approach to initially provide a solid base of the fundamentals of this topic, so that they can be applied later by the teacher. student body.

Figure 1
Percentage of responses to question 1

Likewise, 60.87% of the teachers positively value starting the topic of solving maximums and minimums problems with a problem to which the answer is given as the topic progresses (Figure 2), which means that the Most of the teachers surveyed favor this specific teaching methodology. Showing how some teachers combine theoretical foundations with the practical application of the contents.
In relationship with question 3 (Figure 3), 73.9% of teachers surveyed are in disagreement with the claim that the problems they solve do not relate to their students with daily life neither with a scientific or economic context. This shows that give importance to contextualization of the problems raised and, therefore, the connection with daily life or scientific and economic contexts, seeking to show the relevance and applicability of the problems to students. This situation of posing problems with meaningful contexts for students can make them more motivating, promoting interest and commitment to learning.

82.67% of the teachers surveyed rated question 4 negatively (Figure 4), this implies that the vast majority of teachers tend to develop contextualized problems in their teaching practice. This information agrees with the previous one and suggests that the majority of teachers are committed to the contextualization of maximum and minimum problems, which has certain positive effects. Among them, it helps students apply mathematical concepts to real-life
situations, facilitating the understanding and relevance of the topics. Critical thinking and problem solving are also encouraged by linking mathematical theory with concrete scenarios, in addition to stimulating student interest by presenting them with practical and meaningful challenges.

Figure 4

*Frequency of responses to question 4*

91.3% of teachers surveyed they agree with the statement of question 5, this implies a strong conviction and consensus among teachers about importance of the contextualization in the teaching of these topics (Figure 5). Therefore, it becomes evident that the teachers recognize the relevance of the contextualization in the teaching optimization problems to improve the understanding and interest of the students. It is an indication that among these teachers there is a clear awareness that connecting mathematical concepts with real life situations can facilitate learning and application of the contents. This shared belief in the effectiveness of contextualized problems indicates a trend towards a more oriented teaching application practice and the development of problem-solving skills in a meaningful context.

Figure 5

*Percentage of responses to question 5*
For question 6 (Figure 6), we see discrepancies between the faculty. Thus, 43.4% of teachers surveyed agree with the statement presented, that they mainly follow a text to teach their classes on maximum and minimum problems, this reveals that a significant proportion of teachers tends still depending on large extent of the written materials as the main source of teaching on this issue. Relying excessively on a single resource, such as a textbook, could limit the diversity of teaching approaches and strategies used in the classroom. However, we see how 39.1% of teachers disagree with this statement, showing the opposite trend.

Figure 6
Percentage of responses to question 6

Regarding question 7, 39.13% of the teachers surveyed agree with the statement that they feel more confident using a textbook to explain maximum and minimum problems (Figure 7), this may imply that some of the teachers prefer mainly employ a traditional approach based in texts to address these issues in comparison with other resources or methodologies.

The low percentage of negative responses indicates that many teachers still They may feel more comfortable or confident following a more structured and traditional approach in explaining these concepts following a book, than incorporating other more interactive or innovative methodologies in teaching maximum and minimum problems. This fact indicates that this group of teachers may have a preference for teaching based on textbooks or printed materials to ensure precision and rigor in the communication of content.
73.9% of the teachers disagree that the textbook used in the classroom does not propose maximum and minimum problems that are related to daily life or a context (Figure 8). This shows us again how teachers are generally aware of the relevance of a contextualized approach to teaching maximum and minimum problems, also when they use textbooks.

Regarding the use of the calculator in the classroom, we see that the teachers do not show a clear positive or negative trend, with 39.1% neither agreeing nor disagreeing with this statement (Figure 9).
However, 69.6% of the teachers disagree with the statement I consider it negative that students use the calculator to solve maximum and minimum problems, therefore positioning themselves in favor of the use of the calculator by students (Figure 10).

Below is a table with the means and standard deviations for each of the statements made (Table 2):

**Table 2**

*Means and standard deviations for each question.*

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong></td>
<td>When I explain solving maxima and minima problems, I explain the topic first and then solve problems.</td>
<td>3.87</td>
</tr>
<tr>
<td><strong>2.</strong></td>
<td>When I explain the resolution of maximum and minimum problems, I begin the topic with a problem to which the answer is given as the topic progresses.</td>
<td>3.39</td>
</tr>
<tr>
<td><strong>3.</strong></td>
<td>The problems I solve with my students are not related to daily life or a context.</td>
<td>2.22</td>
</tr>
<tr>
<td><strong>4.</strong></td>
<td>I usually do not create contextualized maximum and minimum problems to solve in the classroom.</td>
<td>2.22</td>
</tr>
<tr>
<td><strong>5.</strong></td>
<td>I am convinced that students understand optimization problems better when they work on problems contextualized to their environment.</td>
<td>4.22</td>
</tr>
</tbody>
</table>
6. I mainly follow a textbook to teach my classes on maximum and minimum problems.  
7. I feel more confident using a textbook to explain maximum and minimum problems.  
8. The textbook that I use in the classroom does not propose maximum and minimum problems that are related to daily life or a context.  
9. I consider it negative that students use the calculator to solve maximum and minimum problems.

### 4 CONCLUSION

This study has provided useful information to identify teaching patterns preferred by a sample of teachers at the Technological University of Santiago – UTESA in relation to teaching calculus. Specifically, it has allowed us to understand some of the predominant educational preferences and practices in relation to solving maximum and minimum problems.

The predisposition of teachers to carry out contextualized maximum and minimum problems has been evidenced to enrich the learning experience of students and improve the understanding and application of mathematical concepts, since connecting mathematics with other disciplines can enrich understanding and learning. However, despite this interest, one aspect that has emerged is that some teachers mainly follow a textbook, which can imply difficulties in adapting the content to the specific needs of the students, affecting the relevance and understanding by students, reducing interactivity in the classroom and limiting opportunities for active participation of students in the learning process. This preference of some teachers to use texts to explain these problems may indicate an inclination towards a more traditional and probably structured approach in teaching these topics, with possible implications for the variety and effectiveness of pedagogical strategies used in the classroom.

On the other hand, it seems that there is no agreement among teachers about the use of software such as the Malmath calculator to solve these types of problems. It is therefore necessary to offer training opportunities to university teachers in innovative and contextualized teaching strategies. Likewise, it will be relevant to promote collaboration between teachers to share good practices and experiences in teaching mathematical problems.

### REFERENCES


